# Moving problem model of rocket on tube type directional part <br> Mô hình bài toán chuyển động của tên lửa trong bộ phận dẫn hướng dạng ống 

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## Keywords:

Uncontrolled rockets; Launcher tube; Oscillation of rockets axis; Semi-links.

## Tì khóa:

Tên lửa không điều khiển; Ống phóng; Dao động của trục tên lửa; Bán liên kết.


#### Abstract

This paper presents the mathematical model to describe the motion of uncontrolled rocket on tube types directional part from fired of the mechanical lost links completely with launcher tube. This model is used to calculate for uncontrolled roket 9M22Y when firing on BM-21 Grad launchers fighting vehicles that our army are researching, designing, manutacturing and improving. This is the scientific basis to study the influence of rocket axis oscillation at this stage to the firing accuracy of uncontrolled rocket.


## Tóm tắt

Bài báo trình bày mô hình toán học mô tả chuyển động của tên lửa không điều khiển trên bộ phận dẫn hướng dạng ống kể từ khi phát hỏa cho đến khi mất liên kết cơ học hoàn toàn với ống phóng. Mô hình được tính toán cho loại đạn phản lực không điều khiển 9 M 22 Y bắn trên dàn phóng xe chiến đấu $\mathrm{BM}-21$ mà quân đội ta đang nghiên cứu thiết kế, chế tạo và cải tiến. Đây là cơ sở khoa học để nghiên cứu ảnh hưởng do dao động của trục tên lửa ở giai đoạn này đến độ chính xác bắn của tên lửa không điều khiển.

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## 1. INTRODUTION

For uncontrolled rocket types, the motion process in launcher tube is extremely important, the motion characteristics of the rocket in this stage directly influence firing accuracy. Therefore, need to have detailed studies about the motion of the rocket in launcher tube, to serve the motion problem of bullet im space, directly study to the dispersion of bullet, to evaluate the firing accuracy of uncontrolled rocket.


Figure 1. The motion stages of rocket in launcher tube
The motion of uncontrolled rocket on tube types directional part includes 2 stage:

- The link motion stage: This stage is calculated from the rocket begin to move until forward centering ring leaves the launcher tube (figure 1a). On this stage, the relativetly motion of rocket with launcher tube includes: Translational motion, rotational motion, the oscillation of rocket axis in launcher tube are limited by gap between centering rings and the inner surface of the launcher tube.
- The semi-link motion stage: This stage is calculated from the forward centering ring leaves mouth launcher tube until aft centering ring leaves the launcher tube. Rocket translational motion and rotation relative links around point A. In fact, there is a gap between the centering ring and launcher tube, so the bullet oscillates in the launcher tube and in any period, the bullet lost link with launcher tube. However, to builds the mathematical model the motion of bullet in the launcher tube, in semi-link motion stage, we assume that: Between bullet and launcher tube always has the link at a single point A located on the aft centering ring and in the vertical plane (figure 1 b ). The oscillation of bullet can be considered in two independent plane (firing plane and horizontal plane). Cause the bullet rotated, quantities on dynamics in the two planes are interchangeable while the gravity is only affects in the firing plane. Therefore, we only need to calculate the problem in the firing plane, this results can be applied in the horizontal plane when without the components of gravity.


## 2. ESTABLISHING DIFFERENTIAL EQUATION SYSTEM TO DESCRIBE THE MOTION OF ROCKET IN THE LAUNCHER TUBE



Figure 2. A diagram of the force effected to the rocket in launcher tube[6]
In order to establish a differential equation system the motion of rocket in the launcher tube, we use assumptions: 1. Rocket, launcher tube is absolutely stiff; 2. The centroid location $O_{T}$ and the mass $m_{T}$ not change when the rocket engine work. 3. The motion of rocket in the launcher tube is uncontrolled, range uncontroled, fixed launcher. 4. Eccentric thrust of the engine is zero, the bullet is static equilibrium and dynamic equilibrium; 5 . The force and torque applied
to the rocket known. With that assumption, the authors have built a diagram of the forces on the rocket in the launcher tube in action Oxyz coordinates are as follows (figure 2) [5].

### 2.1. The forces and moments applied to the rocket when moving in the launcher tube

### 2.1.1. The gravity $G_{T}$

$G_{T}$ is where the gravitational forces acting on the parts of the rocket and put at the centroid of rocket. When the rocket moves in the launcher tube, the rocket engine works, the combustion gases ejected outside, the mass of rocket is reduced and the center position of the rocket also changed. But these changes are negligible for small-size rockets types so can be ignorred. Therefore, when calculating the design of launcher tube, consider $G_{T}$ by constant and the position of centroid is not change.

### 2.1.2. The thrust of rocket engine $P_{T}[3]$

The change law of force $P_{T}$ over time $t$ similar the change law of pressure in the combustion chamber of engine. According [1], the thrust of engine is determined by the following formula:

$$
\begin{aligned}
& P_{T}=0,88 p_{0} F_{t h} n F_{r}(\xi, k)-\text { for winged rocket; } \\
& P_{T}=0,88 p_{0} F_{t h} n F_{r}(\xi, k) \cos \gamma \text { - for turbo rocket. }
\end{aligned}
$$

In there: 0,88 - the loss coefficient of speed and irregularity of air flow through the nozzle; $p_{0}$ - the presure of combustion chamber is determined by interior ballistics problem; $F_{t h}$ - Critical area of the nozzle; $\gamma$ - Tilt angle of the nozzle; n - number of nozzle; $F_{r}(\xi, k)$ - Aerodynamic function depends on enlargement coefficient of nozzle $(\xi)$ and adiabatic exponent $k$.

### 2.1.3. The force and aerodynamic moment [2]

The force and moment that accur due to the resistance of the air. The resistance of the air is determined: The axial aerodynamic force $\mathrm{R}_{\tau}$, the normal aerodynamic force $\mathrm{R}_{\mathrm{n}}$, aerodynamic moment $\mathrm{M}_{\mathrm{a}}$.

- The axial aerodynamic force: $R_{\tau}=C_{\tau} \frac{\rho V_{n}^{2} . S}{2}$;
- The normal aerodynamic force: $\mathrm{R}_{\mathrm{n}}=\mathrm{C}_{\mathrm{n}} \frac{\rho \mathrm{V}_{\mathrm{n}}^{2} \cdot S_{d}}{2}$;
- The aerodynamic moment: $\mathrm{M}_{\mathrm{a}}=\mathrm{R}_{\mathrm{n}} . \mathrm{b}_{\mathrm{a}}$.

In there: $C_{\tau}$ - The coefficient of axial aerodynamic force; $C_{n}$ - The coefficient of normal aerodynamic force ( $C_{n}=1$ ); $\rho$ - Density of air; $S$ - Area cross-section of rocket; $S_{d}$ - Area axialsection of rocket; $V_{n}$-Relative speed between the rocket and the wind; $b_{a}$-Distance from the center of the resistance to centroid of rocket.

### 2.1.4. The friction force

This force accurs due to the friction between directional dowel of the rocket and launcher tube. It is determined by the formula [1]: $F_{m s l}=f_{1} Q_{1} ; F_{m s 2}=f_{2} Q_{2}$.

In there: $f_{1}, f_{2}$ - The friction coefficient between the directional dowel of the rocket and launcher tube; $Q_{1}, Q_{2}$ - Reaction of launcher tube effect to directional dowel.

### 2.1.5. The force of brake mechanism $\boldsymbol{R}_{k}$

The brake mechanism effects on the rocket a holding force $R_{k}$, it hinders the movement of the rocket on the launcher tube. According to [1]: $R_{K} \geq m_{T} \cdot J_{q}$ ( with the rocket 9M22Y fire on the fighting vehicle $\mathrm{BM}-21: R_{k}=6000 \mathrm{~N} \div 8000 \mathrm{~N}$ ). In there: $m_{T}$ - the mass of the rocket; $J_{q}$ - The inertial acceleration. The $\mathrm{R}_{\mathrm{k}}$ forces appear only at the beginning of the launch and by zero when the rocket moves on the launcher tube.

### 2.1.6. The reaction at the directional dowels

The reactions of launcher tube effects on the directional dowel $Q_{1}$ and $Q_{2}$. This force is perpendicular to the axis $O x$ of the moving coordinate system. The forces $Q 1$ and $Q 2$ are solutions of the system of equations:

$$
\left\{\begin{array}{l}
Q_{1}+Q_{2}-G_{T} \cos \varphi=0 \\
Q_{2} l_{2}-Q_{1} l_{1}-R_{K} h_{1}-F_{m s 1} h_{1}-F_{m s 2} h_{2}-M_{a}=0
\end{array}\right.
$$

### 2.2. The equation of translation and rotation of the rocket in the launcher tube

### 2.2.1. The translation equation

$$
\begin{equation*}
m_{T} \cdot \frac{d v}{d t}=P^{\prime}-R_{0}-N(\sin \alpha+f \cdot \cos \alpha) \tag{1}
\end{equation*}
$$

In there: $R_{0}$ - The drag force of initial motion; $R_{0}=\left(F_{m s 1}+F_{m s 2}+R_{k}+R_{\tau}\right) ; P^{\prime}=0,9 . P$ - The thrust of the rocket engine include the losses [1]; $P$ - The thrust of the rocket engine is determined by interior ballistics problem; $f$ - the friction coefficient; $\alpha$ - the tilt angle of twisted slot in the launcher tube; $N$ - the reaction of directional dowel in the twisted slot.

### 2.2.2. The rotation equation

$$
\begin{equation*}
I_{d x} \frac{d \omega}{d t}=M_{q}-M_{0} \pm N \cdot \cos \alpha \cdot \frac{d_{c}}{2}-f \cdot N \cdot \sin \alpha \cdot \frac{d_{c}}{2} \tag{2}
\end{equation*}
$$

In there: $M_{q}$ - the torque due to tilt angle of the nozzle $\gamma ; M_{0}$ - the torque resistance rotational motion initially $M_{0}=\left(h_{1} R_{k}+h_{1} F_{n s 1}+h_{2} F_{m s 2}\right) ; d_{c}$ - the diameter of the rocket; $I_{d x}$ - the axial moment of inertial of the rocket; $\omega$ - the rotational speed of the rocket; $N$ - the reaction of directional dowel in the twisted slot [1]:

$$
N=\frac{\frac{1}{m_{T}}\left(P^{\prime}-R_{0}\right)+\frac{d_{c}}{2 . I_{d x} \cdot \operatorname{tg} \alpha} \cdot M_{0}}{\frac{d_{c}^{2}}{4 . I_{d x} \cdot \operatorname{tg} \alpha}( \pm \cos \alpha-f \cdot \sin \alpha)+\frac{1}{m_{T}}(\sin \alpha+f \cdot \cos \alpha)}
$$

### 2.2.3. The oscillation equation of the rocket axis when moving in the launcher tube

We will study the oscillation of the rocket axis on the vertical plane. All the formulas found in this case can be used for the oscillation on the horizontal plane of the rocket axis if skipped component depends on the gravity.

In the link motion stage, the oscillation of the bullet axis is relatively small and is limited by the gap between the centering ring with the launcher tube. Due to the small gap, the oscillation of rocket axis effect very little to the accuracy of the bullet so it can be ignored .


Figure 3. The coordinate system determines the bullet axis
In semi-link motion stage, the coaxial loss occurs between the rocket and the launcher tube, the bullet moves in the launcher tube with aft centering ring. The axis of the rocket will oscilltate around point A , is located on the aft centering ring in the vertical plane. The motion of point A includes the motion along the launcher tube and fluctuated around the launcher tube axis in the firing plane by the effect of the engine thrust and the gravity. These motion parameters depend on time. Establishhed two coordinate system: The fixed coordinate system $A_{0} X_{0} Y_{0}$ attached with launcher tube, the origin is $A_{0}$ (the position of point A at the initial time), the axis $A_{0} X_{0}$ along the axis of launcher tube, the axis $A_{0} Y_{0}$ perpendicular with $A_{0} X_{0}$. The moving coordinate system: $A X Y$ attached with the rocket, the origin is $A$, the axis $A X$ direction along the axis of the rocket, the axis AY perpendicular AX in the firing plane (figure 3). The motion of the point A is the motion of the rocket in the launcher tube. The oscillation of rocket is determined by the angle $\beta$, the positive direction of this angle is the way counterclockwise. For convenience when surveying, considering AC coincides with the axis of the rocket.

The force and moment applied to the bullet when moving in the launcher tube [4]:

+ The moment: $M_{q}=G_{T} l_{A C} \cos \left(\theta_{0}+\beta\right)$;
+ The inertial force along the axis $A_{0} X_{0}: F_{q t X}=m_{T} \ddot{X}_{C}$;
+ The inertial force along the axis $A_{0} Y_{0}: F_{q t Y}=m_{T} \ddot{Y}_{C}$.
With conditions above, the oscillation equation of the bullet axis:

$$
\begin{equation*}
J_{A} \ddot{\beta}=m_{T} \ddot{X}_{c} l_{A C} \sin \beta-m_{T} g l_{A C} \cos \left(\theta_{0}+\beta\right)-m_{T} \ddot{Y}_{C} l_{A C} \cos \beta \tag{}
\end{equation*}
$$

In there, $J_{A}$ - Inertial moment of the bullet around point $A: J_{A}=J_{C}+m_{T} l_{A C}^{2} ; J_{\mathrm{C}}$ - Inertial moment of the rocket around point $\mathrm{C} ; l_{A C}$ - Distance from the centroid of rocket to aft centering ring; Due to $\beta$ is small: $\sin \beta \approx \beta ; \cos \beta \approx 1$.

Ignore friction, equation of translation motion of the rocket along the axis $A_{0} X_{0}, A_{0} Y_{0}$ from loss link between forward centering ring and launcher tube:

$$
\begin{aligned}
& m_{T} \ddot{X}_{C}=\left[P-m_{T} g \sin \left(\theta_{0}+\beta\right)\right] \cos \beta \\
& m_{T} \ddot{Y}_{C}=\left[P-m_{T} g \sin \left(\theta_{0}+\beta\right)\right] \sin \beta
\end{aligned}
$$

Equation ( ${ }^{*}$ ) becomes: $\ddot{\beta}=-m_{T} g l_{A C}\left(\cos \theta_{0}-\beta \sin \theta_{0}\right) / J_{A}$
From (1), (2), (3) we have a system of equation describing the motion of rocket in the launcher tube:

$$
\left\{\begin{array}{l}
m_{T} \frac{d v}{d t}=P^{\prime}-R_{0}-N(\sin \alpha+f \cdot \cos \alpha)  \tag{4}\\
I_{d x} \frac{d \omega}{d t}=M_{q}-M_{0} \pm N \cdot \cos \alpha \cdot \frac{d_{c}}{2}-f \cdot N \cdot \sin \alpha \cdot \frac{d_{c}}{2} \\
\frac{d Y_{1}}{d t}=\dot{\beta}=Y_{2} \\
\frac{d Y_{2}}{d t}=\ddot{\beta}=-m_{T} g l_{A C}\left(\cos \theta_{0}-\beta \sin \theta_{0}\right) / J_{A} .
\end{array}\right.
$$

Initial condition of equations system: $t=t_{0} ; v=v\left(t_{0}\right) ; \omega=\omega\left(t_{0}\right) ; Y_{1}=Y_{1}\left(t_{0}\right) ; Y_{2}=Y_{2}\left(t_{0}\right)$.

## 3. THE SOLUTION AND DISCUSSIONS

Systems of differential equations (4) was solve by numerial integration method, using algorithm Runge - Kutta 4. We get the following results (figure 4).

a. Displacement of the rocket centroid

c. Rotation angle of the rocket around the axis of launcher tube

e. Deviation angle of the rocket axis in semi-link motion stage

b. Velocity of the rocket centroid

d. Angular velocity of the rocket around the axis of launcher tube

f. Deviation angular velocity of the rocket axis in semi-link motion stage

Figure 4. Results solve bullet motion 9M22Y in the launcher tube

From the results of graphs 4 . We have come to the conclusions below:

- The bullet speed increased rapidly and to reach the maximum value in the mouth launcher tube (figure 4b). Survey results are suitable with reality;
- The deviation angle and the deviation angular velocity gradually increases in the semilink motion stage (figure $4 \mathrm{e}, 4 \mathrm{f}$ ). The value of deviation angle and deviation angular velocity are relatively small;
- In the semi-link motion stage, the bullet axial oscillation causes bounce angle. This is the nutation angle initial of the bullet, so it directly affects the firing accuracy when launching;
- If the oscillation of the launcher is controlled, the gravity is the only sourse of bullet axis oscillation when the bullet leaves mouth launcher tube. To limit this effect, the design of the bullet need to pay attention to composition and structure coefficient that ensure optimal value of moment of inertia $J_{A}=J_{C}+m l_{A C}^{2}$.


## 4. CONCLUSION

The paper has built a physical model and mathematical model describing the motion of uncontrolled rocket on tube type directional part. The paper results is input to the problem of the rocket motion in space, serve for the design, manufacture, improvement and repair, calculate the dispersion and firing accuracy of uncontrolled rocket when launching. Besides, the dispersion of uncontrolled rocket is greatly influenced by the condition of launch, so research results also have important significance in the firing correction or provide technical solutions to reduce dispersion of the bullet when launching.

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## REFERENCES

[1]. Nguyễn Xuân Anh, Nguyễn Lạc Hồng, (1998). Giáo trình tính toán bệ phóng và động lục học khi phóng, Học viện KTQS.
[2]. Nguyễn Xuân Anh, (2000). Động lưc học bệ tên lưa, NXB Quân đội Nhân dân.
[3]. Phạm Thế Phiệt,(1995). Lý thuyết động cơ tên lưa, Học viện KTQS.
[4]. Đỗ Sanh, (2008). Cơ học giải tich, NXB ĐH Bách khoa, Hà Nội.
[5]. Nguyễn Thanh Hải, Nguyễn Thái Dũng, Võ Văn Biên, (2016). Trang bị vũ khí phản lục, NXB Quân đội Nhân dân.
[6]. Nguyễn Duy Phồn, (2017). Luận án tiến sĩ, Học viện Kỹ thuật Quân sự.

