Calculating the launcher tube thickness of a combined weapon model
Tính toán bề dày ống phóng cho một mô hình vũ khí kết hợp

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Abstract
This article presents the optimal solution of optimizing launcher-tubes’ thickness for a combined weapon model. The effect of mechanical loading and influence of temperature is taken into the optimal tube thickness problem. The considered model is applied to the decoy rocket with warhead consisting of 6 modules. The results were compared to the actual thickness of launcher tube to examine the accuracy of the considered model. They also serve as the basis for researching related issues and completing the designs of decoy rocket tube.

Tóm tắt
Mục đích của nghiên cứu là tối ưu hóa chiều dày ống phóng của tổ hợp phóng loạt có kết cấu kiểu mô đun chịu tác dụng của lực khí thuốc có tính đến ảnh hưởng của nhiệt độ. Áp dụng khảo sát với mô hình tổ hợp phóng loạt cao thấp áp (CTA) có vật phóng gồm 6 mô đun. Kết quả khảo sát được so sánh với bề dày thực tế của ống phóng để đánh giá được tính chính xác của mô hình khảo sát; đồng thời là cơ sở để nghiên cứu các bài toán khác và hoàn thiện cho thiết kế ống phóng đạn nhiễu.

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1. INTRODUCTION
In the field of military, the maneuverability of the equipment is one of the key factors affected to the success of mission. In this way, the research, design and optimization of equipment structures play as an important role in ensuring that the equipments are sufficiently durable, reasonable in size and cost effective material. The new concept weapon operated on principle of high/low pressure chambers combined with mortars is one of the best answers in the problem mentioned above. During the firing, the launcher-tube is impacted by external loading from the burning products and environment. This effect causes the change in the mechanical properties of the launcher-tube material.
Nowadays, the effect of thermal and mechanical loading on the launcher-tube using principle of high-low pressure chambers or mortars are mentioned in Refs [4, 6]. In these works, the influence of extra propellant charge is not taken into the calculating process [1, 4, 6]. In addition, the calculation process only uses the maximum value of mechanical load and ignores the effect of thermal load [7]. These lead to the results of problem will not reflect the accuracy of the mentioned model. In other hand, the use of computer-aided modern methodology to develop the thickness optimization software has not been studied and mentioned in the design problems of the launcher-tube. Therefore, the thickness optimization problem is chosen to determine the reasonable thickness of the launcher tube in the design and manufacture of the equipment in order to ensure the above mentioned purposes.

2. THEORETICAL BASIS

2.1. The interior ballistics of combined weapon model

The schematic presentation of model is shown in Figure 1.

![Fig 1. Schematic diagram of model](image)

1- High pressure chamber. 2- Propellant. 3- Nozzle. 4- Piston and extra propellant. 5- Projectiles. 6- Tube

The system of equations which describes physical processes in the mentioned model can be written in the form:

\[
\begin{align*}
\frac{dz_1}{dt} &= \frac{p_c}{I_{11}}, \quad \frac{d\psi_1}{dt} = \chi_1(1 + 2\lambda z_1 + 3\mu z_1) \frac{dz_1}{dt}; \\
\frac{d\eta_1}{dt} &= \frac{\varphi_{21} K_0 S_{th} (p_c - p_1)}{\omega_1 \sqrt{\chi_{n1} f_1 \tau_c}} \\
p_c &= \frac{f \tau_c \omega_1 (\psi_1 - \eta_1)}{W_{oc} - \frac{\omega_1}{\delta_1} (1 - \psi_1) - \alpha_1 \omega_1 (\psi_1 - \eta_1)}; \\
\frac{d\tau_c}{dt} &= \frac{(1 - \tau_c) \frac{d\psi_1}{dt} + (1 - k) \tau_c \frac{d\eta_1}{dt}}{\psi_1 - \eta_1}; \\
\frac{dz_2}{dt} &= \frac{p_i}{I_{12}}, \quad \frac{d\psi_2}{dt} = \chi_2(1 + 2\lambda z_2 + 3\mu z_2) \frac{dz_2}{dt}; \\
\frac{d\eta_2}{dt} &= \frac{\varphi_{22} K_{th} S_{th} (p_i - p_{th})}{\omega_2 \sqrt{\chi_{n2} f^* \tau_i}} \\
p_i &= \frac{f^* \tau_i (\omega_1 \eta_1 + \omega_2 \psi_2 - \omega \eta_2)}{W_{oi} - \alpha_2 (\omega_1 \eta_1 + \omega_2 \psi_2 - \omega \eta_2) - \frac{\omega_2}{\delta_2} (1 - \psi_2) + S_{oi} l}; \\
\frac{dv}{dt} &= \frac{S_{oi} P_i}{\varphi m} \\
\frac{d\tau_i}{dt} &= \frac{(k \tau_c - \tau_i) \omega_1 \frac{d\eta_1}{dt} + (1 - \tau_i) \omega_2 \frac{d\psi_2}{dt} - \omega \tau_i \frac{d\eta_2}{dt} - \frac{\theta m}{f^* \psi} \frac{dv}{dt}}{\omega \eta_1 + \omega_2 \psi_2 - \omega \eta_2} \\
\frac{d}{dt} &= v
\end{align*}
\]

The received results are the functions of the pressure and temperature in low pressure chamber over time.
2.2. The stress problem of launcher-tube subjected to mechanical and thermal loading

Consider the launcher-tube limited by two diameters $d_t, d_n$, and the length $l$ in Figure 2. The loads acting on the tube are determined by solving the Equations (1).

![Launcher-tube model](image)

Due to the symmetrical of launcher-tube, the problem can be investigated with a quarter of tube. The meshing model is shown in Figure 3.

![Meshing model of a quarter of tube](image)

By using FEM, the relationship between element displacements and nodal displacements are represented as follows:

$$\{u\}_e = [N]\{q\}_e$$

(2)

The strain-displacement relations are expressed in the form

$$\{\varepsilon\}_e = [\varepsilon]\{u\}_e = [\varepsilon][N]\{q\}_e = [B]\{q\}_e$$

(3)

with [B] - the element deformation matrix (the derivative operator matrix).

The stress of element can be defined according to Hook’s law as:

$$\{\sigma\}_e = [D]\{\varepsilon\}_e = [D][B]\{q\}_e$$

(4)

[D] - the material property matrix.

Using the principle of virtual displacements, we get the expression:

$$\int_{S_e} \{\varepsilon\}_e^T \{\sigma\}_e dS = \int_{S_e} \{\delta u\}_e^T \{p\}_e dS$$

(5)

Replace Equation (4) in Equation (5) received the result

$$\int_{S_e} \{\varepsilon\}_e^T [D][B]\{q\}_e dS = \int_{S_e} \{\delta u\}_e^T \{p\}_e dS$$

(6)
From Equations (2) and (3), we obtain

\[
\{\delta e\}_e = [B]\{\delta q\}_e \quad \text{hay} \quad \{\delta e\}_e^T = [B]^T\{\delta q\}_e^T
\]  

(7)

\[
\{\delta u\}_e^T = [N]^T\{\delta q\}_e^T
\]  

(8)

Substituting Equations (7) and (8) into Equation (6). By transformations, Equation (6) can be rewritten in the following form:

\[
\{\delta q\}_e^T \left( \int [B]^T[D][B]dS \right)\{q\}_e = \{\delta q\}_e^T \left( \int [N]^T\{p\}_e dS \right)
\]  

(9)

Equation (9) can be expressed in short form as:

\[
[K]_e\{q\}_e = \{P\}_e
\]  

(10)

with:

\[
[K]_e = \int [B]^T[D][B]dV \quad \text{- the stiffness matrix of element.}
\]

\[
\{P\}_e = \int [N]^T\{p\}_e dS \quad \text{- element load vector.}
\]

The matrix [B] may be written as:

\[
[B] = \begin{bmatrix}
\frac{\partial N_1(x)}{\partial x} & 0 & \frac{\partial N_2(x)}{\partial x} & 0 & \frac{\partial N_3(x)}{\partial x} & 0 & \frac{\partial N_4(x)}{\partial x} & 0 \\
0 & \frac{\partial N_1(x)}{\partial y} & 0 & \frac{\partial N_2(x)}{\partial y} & 0 & \frac{\partial N_3(x)}{\partial y} & 0 & \frac{\partial N_4(x)}{\partial y} \\
\frac{\partial N_1(x)}{\partial x} & \frac{\partial N_1(x)}{\partial y} & \frac{\partial N_2(x)}{\partial x} & \frac{\partial N_2(x)}{\partial y} & \frac{\partial N_3(x)}{\partial x} & \frac{\partial N_3(x)}{\partial y} & \frac{\partial N_4(x)}{\partial x} & \frac{\partial N_4(x)}{\partial y} \\
\end{bmatrix}
\]  

(11)

The components in the matrix B is defined by [8]. From (11) the matrix [K] are calculated, then we obtained:

\[
[K]_e = \int \int (B^e)^T DB^e \det|J|d\xi d\eta
\]  

(12)

For the whole body, we can establish the stiffness matrix [K] and the overall load vector {P}. Thus, Equation (10) can rewritten by

\[
[K]\{u\} = \{P\}
\]  

(13)

The displacement of element {u} can be calculated by solving Equation (13). Then the stress of element {\sigma} are determined from Equation (4).

2.3. The problem of thickness optimization of launcher tube

The structural optimization is the problem that aims to determine the reasonable geometric dimensions of structure to satisfy the given constraints. The target function usually denotes
quantities that need to be minimized such as weight, volume, and material cost. Constraints are usually conditions of durability, hardness or other conditions. With the proposed model, for simplicity, we consider the optimization of launcher tube thickness relative to the structural strength. The problem is posed as follows:

Find the optimal radius vector \( \{r\} = \{r_1, r_2, \ldots, r_n\}^T \) to minimize the cost function:

\[
f(r) = r - r_t
\]  

(14)

And the path constraints:

\[
\begin{align*}
g(x) & \leq [\sigma] \\
r_{\text{min}} & \leq r \leq r_{\text{max}}
\end{align*}
\]  

(15)

In this setting:  
\( f(r) \) - target function  
\( r \) - outer radius of launcher-tube (optimal variable).  
\( r_t \) - inner radius of launcher-tube (constant).

The solution of problem could be found by numerical method. The Sequential Quadratic Programming (SQP) method has been used to solve this problem. The SQP method can be reference in [2, 5, 9]. The received results are the thickness values at the cross sections along the length of the tube.

3. RESULTS AND DISCUSSION

The survey model is applied to the launcher tube of decoy rocket with warhead consist of six modules. The procedures are built under the Matlab software. The geometrical and material parameters are summarized in Table 1. The interior ballistics parameters can be found in [2].
Table 1. Geometrical and material data of the launcher-tube

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>1109.6 mm</td>
</tr>
<tr>
<td>Inner diameter</td>
<td>110 mm</td>
</tr>
<tr>
<td>Outer diameter</td>
<td>120 mm</td>
</tr>
<tr>
<td>Young’s modulus</td>
<td>$7.1 \times 10^3$ MPa</td>
</tr>
<tr>
<td>Poission’s ratio</td>
<td>0.34</td>
</tr>
</tbody>
</table>

3.1. Interior ballistics results of combined weapon model.

The results are shown in Figure 5 and Table 2. The maximum pressure in the launcher-tube $P_{\text{tmax}} = 48.25$ (kG/cm$^2$) at time $L = 4.99 \times 10^{-1}$ cm and the maximum temperature in the launcher-tube $T_{\text{tmax}} = 2394.94$ (ºK) at time $L = 2.7 \times 10^{-2}$ cm.

![Fig. 5. The simulation curve of pressure (a) and temperature (b) in the launcher-tube](image)

Table 2. The pressure and temperature of gas at the special time

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Time</th>
<th>$T_1$ get max value</th>
<th>$P_1$ get max value</th>
<th>Grenade out of the tube</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure (kG/cm$^2$)</td>
<td>26.16</td>
<td>48.25</td>
<td></td>
<td>1.88</td>
</tr>
<tr>
<td>Temperature (ºK)</td>
<td>2394.94</td>
<td>2231.41</td>
<td>1951.48</td>
<td></td>
</tr>
</tbody>
</table>

3.2. The results of strain tests

The survey model is affected by the symmetry of the load, so we can investigate for a quarter of the circle with the symmetry boundary. The problem can be calculated in two case: pressure reaches maximum value and temperature reaches maximum value. The results are shown in Figure 6 and Table 3.
Fig. 6. The graphs of Von Mises strain at the special time
a- Pressure get maximum value (P_{\text{max}})  
b- Temperature get maximum value (T_{\text{max}})

Table 3. The values of strain at the special time

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Time</th>
<th>( T_t ) get max value</th>
<th>( P_t ) get max value</th>
<th>Grenade out of the tube</th>
</tr>
</thead>
<tbody>
<tr>
<td>Von Mises Strain</td>
<td></td>
<td>3.91 \times 10^7</td>
<td>5.71 \times 10^7</td>
<td>2.22 \times 10^6</td>
</tr>
</tbody>
</table>

3.3. The results of launcher-tube thickness optimization

By solving the problem, the results of optimal launcher-tube thickness are given in Table 4.

Table 4. Optimal thickness results in sections

<table>
<thead>
<tr>
<th>Waypoint (cm)</th>
<th>Parameters</th>
<th>6.3 \times 10^{-6}</th>
<th>8.3 \times 10^{-5}</th>
<th>2.7 \times 10^{-3}</th>
<th>9.7 \times 10^{-3}</th>
<th>4.99 \times 10^{-1}</th>
<th>3.2</th>
<th>15.4</th>
<th>110.96</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pressure (kG/cm²)</td>
<td>3.58</td>
<td>14.15</td>
<td>26.16</td>
<td>39.94</td>
<td>48.25</td>
<td>32.12</td>
<td>11.37</td>
<td>1.88</td>
</tr>
<tr>
<td></td>
<td>Difference of temperature (ºK)</td>
<td>25*2</td>
<td>62*8</td>
<td>195.1</td>
<td>446.3</td>
<td>740.6</td>
<td>829.3</td>
<td>654.7</td>
<td>315.4</td>
</tr>
<tr>
<td></td>
<td>Inner radius (cm)</td>
<td>5.5</td>
<td>5.5</td>
<td>5.5</td>
<td>5.5</td>
<td>5.5</td>
<td>5.5</td>
<td>5.5</td>
<td>5.5</td>
</tr>
<tr>
<td></td>
<td>Outer radius (cm)</td>
<td>5.71</td>
<td>5.783</td>
<td>5.966</td>
<td>5.998</td>
<td>5.996</td>
<td>5.996</td>
<td>5.989</td>
<td>5.974</td>
</tr>
<tr>
<td></td>
<td>Actual thickness (cm)</td>
<td>0.21</td>
<td>0.283</td>
<td>0.466</td>
<td>0.483</td>
<td>0.498</td>
<td>0.496</td>
<td>0.489</td>
<td>0.474</td>
</tr>
</tbody>
</table>

Discussions:

- During the firing, the pressure and temperature of gas tended to increase rapidly near the bottom of the barrel and decreased gradually along the axial length. The value of strain at the time pressure get maximum was higher than the value of strain at the time temperature get maximum. Due to effect of thermal load be taken into the calculating of the durability of the tube.

- The optimum thickness reached maximum value at the position of largest mechanical load and got minimum value at the bottom of launcher tube. The change in thickness increased from the bottom to the position of pressure reached maximum value and decreased along the length of the launcher tube.
- The optimal thickness results are examined by comparing the actual thickness. In this case, the optimal thickness values are smaller and approximate to the actual thickness. Thus, the optimum launcher tube thickness results for the survey model ensure reliability.

4. CONCLUSIONS

The calculation of launcher-tube thickness operated on principle of high/low pressure chambers combined with mortars has been studied. The interior ballistics model was proposed and the influence of thermal mechanical loading on the launcher-tube was analyzed. The author has developed the program for the optimal launcher-tube thickness problem with constraints on durability. Application is considered on the decoy rocket with warhead consists of six modules. The optimum thickness are approximate to the actual thickness of launcher tube. Thus, the results and calculation program ensure reliability.

The model is only investigated in the case launcher tube is subjected to structural strength constraint. The results of problem is useful in designing and testing the durability of the launcher-tube. Therefore, it is necessary to continue to study and investigate with various constraints in order to calculating, researching, designing of the launcher-tubes according to the above principle model. The program may be used to investigate for the gun with smooth barrel types.

REFERENCES

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[7]. Nguyễn Thế Lực, 2015. Tính toán, thiết kế, chế tạo cơ cấu tách rải nhiễu nhiều của đạn DN10.VN-1 cho tàu Hải quân, LVThS, Học viện KTQS.