OPTIMUM ABSORBER PARAMETERS FOR ROTATING SHAFTS WITH VARIABLE ANGULAR VELOCITY

CÁC THÔNG SỐ TỐI ƯU CỦA BỘ HẤP THỤ ĐỘNG LỰC CHO CÁC TRỤC QUAY VỚI VẬN TỐC GÓC THAY ĐỔI

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ABSTRACT

The rotating shaft is used to transmit torque from a driving device, such as a motor or engine. The rotating shaft can carry pulleys, gears, etc to transmit rotary motion via belts, and chains, mating gears. But the shaft is not always rotating at constant angular velocity but due to unstable current or due to sudden acceleration or deceleration. The rotating shaft with variable angular velocity. Therefore, this paper studies to determine the optimal parameter of the tuned mass damper to reduce the torsional vibration for the rotating shaft with variable angular velocity by using the maximization of equivalent viscous resistance method.

Keywords*: The rotating shaft, torsional vibration, equivalent viscous resistance, optimum absorber parameter.*

TÓM TẮT

Trục quay được sử dụng để truyền mô-men xoắn từ thiết bị truyền chuyển động, chẳng hạn như motor hoặc động cơ. Trục quay có thể mang ròng rọc, bánh răng,... để truyền chuyển động quay qua dây đai, dây xích hoặc bánh răng phối ghép. Nhưng không phải lúc nào trục cũng quay với vận tốc góc không đổi mà do dòng điện không ổn định hoặc do tăng giảm tốc đột ngột. Trục sẽ quay với vận tốc góc thay đổi. Do đó, bài báo này trình bày nghiên cứu xác định các tham số tối ưu của bộ hấp thụ động lực để giảm dao động xoắn cho trục quay có vận tốc góc biến đổi theo phương pháp cực đại lực cản nhớt tương đương.

Từ khóa*: Trục quay, dao động xoắn, lực cản nhớt tương đương, tham số tối ưu của bộ hấp thụ động lực.*

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1. INTRODUCTION

Absorber is a tuned-mass damper (TMD), or dynamic vibration absorber (DVA), is found to be an efficient, reliable, and low-cost suppression device for the technical constructions and mechanical devices [1-10]. In [5-8] studied to find the optimal parameter of the DVA to reduce torsional vibration for the shaft. When designing absorbers to reduce vibration for the main system, the shape of the absorbers is quite rich, depending on the type of structure to be installed. So, in [1-4] studied and determined the optimal parameters of the TMD to reduce torsional vibration for the shaft. However, the studies in references [1-8] only considered the rotating shaft with constant angular velocity.

To the best knowledge of the authors, there have been no studies based on the maximum equivalent viscous resistance method to determine the optimum parameters of the TMD for the rotating shaft with variable angular velocity. So, to overcome the limitations and develop the research results in references [1-4]. In this paper, the author continues to find the optimal parameters of the TMD to reduce torsional vibration for the shaft, in which the rotating shaft with variable angular velocity by using the maximum equivalent viscous resistance method according to the reference [9].

2. SHAFT MODELLING AND VIBRATION EQUATIONS

Figure 1 shows a pendulum type TMD attached to a shaft. The symbols of the shaft and TMD are summarized in Appendix.

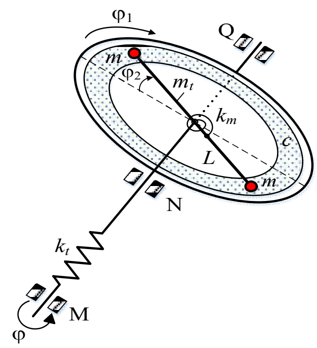

Figure 1. Shaft model attached with a TMD

From [4], we have

$$\begin{aligned}&[3M\rho^2 + 2(m_tL^2 + 3mL^2)]\ddot{\varphi}_1\\&+2(m_tL^2 + 3mL^2)\ddot{\varphi}_2 + 3k_t(\varphi_1 - \varphi) - 3M(t) = 0\end{aligned} \quad (1)$$

$$(2m_tL^2 + 6mL^2)\ddot{\varphi}_1 + 2(2m_tL^2 + 6mL^2)\ddot{\varphi}_2 + 6cL^2\dot{\varphi}_2 + 3k_m\varphi_2 = 0 \quad (2)$$

In this paper, the author considers the cause of torsional vibration for the system is because the rotating shaft with variable angular velocity, so we have:

$$\begin{cases}\varphi_1 - \varphi(t) = \theta \rightarrow \varphi_1 = \theta + \varphi(t);\ \ \dot{\varphi}_1 = \dot{\theta} + \dot{\varphi}(t);\ \ \ddot{\varphi}_1 = \ddot{\theta} + \ddot{\varphi}(t)\\ M(t) = 0\end{cases} \quad (3)$$

Introduce the parameters

$$\mu=\frac{m}{M},\mu_t=\frac{m_t}{M},\ \omega_D^2=\frac{k_t}{M\rho^2},\ \omega_d^2=\frac{3k_m}{2(3m+m_t)L^2},$$
$$\xi=\frac{3c}{2(3m+m_t)\omega_d},\ \omega_d=\alpha\omega_D,\ L=\gamma\rho,\ \omega=\beta\omega_D \tag{4}$$

Substituting Eqs. (3, 4) into Eqs. (1, 2). The matrix equation of the system can be rewritten as

$$\begin{bmatrix}\dot\theta\\ \dot\varphi_2\\ \ddot\theta\\ \ddot\varphi_2\end{bmatrix}=\begin{bmatrix}\Psi_{11}&\Psi_{12}&\Psi_{13}&\Psi_{14}\\ \Psi_{21}&\Psi_{22}&\Psi_{23}&\Psi_{24}\\ \Psi_{31}&\Psi_{32}&\Psi_{33}&\Psi_{34}\\ \Psi_{41}&\Psi_{42}&\Psi_{43}&\Psi_{44}\end{bmatrix}\begin{bmatrix}\theta\\ \varphi_2\\ \dot\theta\\ \dot\varphi_2\end{bmatrix}+\begin{bmatrix}0\\0\\-1\\0\end{bmatrix}\ddot\varphi(t) \tag{5}$$

in which

$$\Psi_{11}=0;\ \Psi_{21}=0;\ \Psi_{31}=-\omega_D^2;\ \Psi_{41}=\omega_D^2;\ \Psi_{12}=0;\ \Psi_{22}=0;$$
$$\Psi_{32}=\frac{2(3\mu+\mu_t)\gamma^2\alpha^2\omega_D^2}{3};\ \Psi_{42}=-\frac{(3+6\mu\gamma^2+2\mu_t\gamma^2)\alpha^2\omega_D^2}{3};$$
$$\Psi_{13}=1;\ \Psi_{23}=0;\ \Psi_{33}=0;\ \Psi_{43}=0;\ \Psi_{14}=0;\ \Psi_{24}=1; \tag{6}$$
$$\Psi_{34}=\frac{4(3\mu+\mu_t)\xi\gamma^2\alpha\omega_D}{3};\ \Psi_{44}=-\frac{2(3+6\mu\gamma^2+2\mu_t\gamma^2)\xi\alpha\omega_D}{3}$$

3. DETERMINATION OF OPTIMAL PARAMETERS OF THE TMD

After short modification the Eqs. (1-3), we obtain

$$M\rho^2\ddot\theta+k_t\theta=k_m\varphi_2+2cL^2\dot\varphi_2-M\rho^2\ddot\varphi(t) \tag{7}$$

So, the torque equivalent of the TMD set on the rotating shaft is

$$M_{eqv}=k_m\varphi_2+2cL^2\dot\varphi_2 \tag{8}$$

According to [9], the equivalent resistance coefficient of the TMD on the primary structure is obtained as

$$c_{eqv}=-\frac{\left\langle M_{eqv}\dot\theta\right\rangle}{\left\langle\dot\theta^2\right\rangle} \tag{9}$$

Substituting Eq. (8) into Eq. (9), this becomes

$$c_{eqv}=-\frac{2cL^2\left\langle\dot\varphi_2\dot\theta\right\rangle+k_m\left\langle\varphi_2\dot\theta\right\rangle}{\left\langle\dot\theta^2\right\rangle} \tag{10}$$

If the primary system is excited by the angular acceleration of the rotating shaft, $\ddot\varphi(t)$, is assumed as white noise, has the spectral density Y_a, then the average value of Eq. (10) are the components of the matrix $\mathbf{\Delta}$ in Eq. (12). We have

$$c_{eqv}=-\frac{2cL^2\Delta_{34}+k_m\Delta_{32}}{\Delta_{33}} \tag{11}$$

Matrix $\mathbf{\Delta}$ is a solution of Lyapunov algebraic equation in Eq. (12).

$$\mathbf{\Psi\Delta}+\mathbf{\Delta\Psi}^T+Y_a\mathbf{Z}_a\mathbf{Z}_a^T=\mathbf{0} \tag{12}$$

where

$$\mathbf{\Psi}=\begin{bmatrix}\Psi_{11}&\Psi_{12}&\Psi_{13}&\Psi_{14}\\ \Psi_{21}&\Psi_{22}&\Psi_{23}&\Psi_{24}\\ \Psi_{31}&\Psi_{32}&\Psi_{33}&\Psi_{34}\\ \Psi_{41}&\Psi_{42}&\Psi_{43}&\Psi_{44}\end{bmatrix},\ \mathbf{Z}_a=\begin{bmatrix}0\\0\\-1\\0\end{bmatrix} \tag{13}$$

Substituting Eq. (13) into Eq. (12), the matrix **Δ** can be determined as:

$$\mathbf{\Delta}=\begin{bmatrix}\Delta_{11}&\Delta_{12}&\Delta_{13}&\Delta_{14}\\ \Delta_{21}&\Delta_{22}&\Delta_{23}&\Delta_{24}\\ \Delta_{31}&\Delta_{32}&\Delta_{33}&\Delta_{34}\\ \Delta_{41}&\Delta_{42}&\Delta_{43}&\Delta_{44}\end{bmatrix} \tag{14}$$

in which

$$\Delta_{32}-\frac{3[(\mu+\frac{1}{3}\mu_t)\gamma^2+\frac{1}{2}]^2Y_a}{(3\mu+\mu_t)\gamma^2\omega_D^2} \tag{15}$$

$$\Delta_{33}=\frac{3Y_a\left[\begin{array}{l}\frac{1}{8}+(\frac{1}{2}+(\mu+\frac{1}{3}\mu_t)\gamma^2)^3\alpha^4\\ +2(\xi^2(\mu+\frac{1}{3}\mu_t)\gamma^2+\frac{1}{2}\xi^2-\frac{1}{4})(\frac{1}{2}+(\mu+\frac{1}{3}\mu_t)\gamma^2)\alpha^2\end{array}\right]}{(3\mu+\mu_t)\gamma^2\xi\alpha\omega_D} \tag{16}$$

$$\Delta_{34}=\frac{1}{72}\frac{Y_a\left[(2\mu_t\gamma^2+6\mu\gamma^2+3)^2\alpha^2-9\right]}{(\mu+\frac{1}{3}\mu_t)\gamma^2\xi\alpha\omega_D} \tag{17}$$

Using Eqs. (4, 11, 15-17) gives

$$c_{eqv}=\frac{-8(\frac{1}{2}m+\frac{1}{6}m_t)\omega_D\alpha\xi\gamma^2\rho^2}{\left[\begin{array}{l}\alpha^4\gamma^4\mu^2(8\gamma^2(\mu+\frac{1}{3}\mu_t)+12)+\alpha^2\gamma^2(\mu+\frac{1}{3}\mu_t)\\ (16\gamma^4(\mu+\frac{1}{3}\mu_t)\xi^2+16\xi^2+6\alpha^2-4)+\alpha^4\\ +4\alpha^2\xi^2-2\alpha^2+1\end{array}\right]} \tag{18}$$

Maximization of the equivalent resistance coefficient are expressed as

$$\left.\frac{\partial c_{eqv}}{\partial\alpha}\right|_{\alpha_{opt}^{MEVR}=\alpha}=0 \tag{19}$$

$$\left.\frac{\partial c_{eqv}}{\partial\xi}\right|_{\xi_{opt}^{MEVR}=\xi}=0 \tag{20}$$

Combining Eq. (18) with Eqs. (19-20), we obtain the optimal parameters as follows

$$\alpha_{opt}^{MEVR} = \frac{3}{2\mu_t\gamma^2 + 6\mu\gamma^2 + 3} \quad (21)$$

$$\xi_{opt}^{MEVR} = \frac{\gamma\sqrt{3\mu+\mu_t}}{\sqrt{6+12\mu\gamma^2+4\mu_t\gamma^2}} \quad (22)$$

Eqs. (21, 22) represent the optimal parameters of the TMD to reduce the torsional vibration of the rotating shaft with variable angular velocity by using the maximization of equivalent viscous resistance method.

4. NUMERICAL SIMULATION

To evaluate the reliability of the optimal parameters are determined by Eqs. (21, 22). The author simulates the vibration of the system with the input parameters of the rotating shaft and TMD are given in Table 1.

Table 1. The input parameters of the rotating shaft and TMD

Parameter	M	ρ	k_t	m_t	m	L	μ	γ
Value	450kg	1.2m	10^6Nm/rad	14kg	11kg	1.0m	0.035	0.83

From Eqs. (4, 21, 22) and Table 1, we infer the optimal parameters of the TMD in Table 2.

Table 2. The optimal parameters of the TMD

Parameter	α_{opt}^{MEVR}	ξ_{opt}^{MEVR}	c	k_m
Value	0.954	0.107	126.08Ns/m	43996.26Nm/rad

4.1. Case 1: Initial torsional vibration $\theta_0 = 0.03$(rad)

From the parameters in Tables 1 to 2 and case 1, using Maple software simulates the torsional vibration of the rotating shaft is shown in Figure 2.

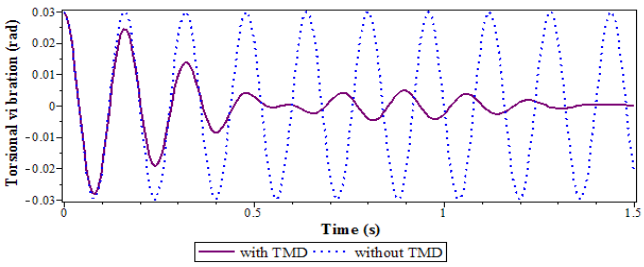

Figure 2. The vibration of the shart in the case with initial torsional vibration θ_0=0.03(rad)

4.2. Case 2: Initial torsional vibration θ_0 = 0(rad)and initial angular velocity of $\dot{\theta}_0 = 1.0$ (rad/s)

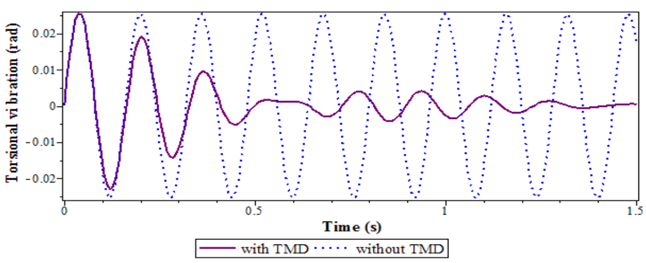

Figure 3. The vibration of the shart in the case with initial angular velocity of $\dot{\theta}_0 = 1.0$ (rad/s)

From the parameters in Tables 1, 2 and case 2, Maple software is usedto simulate the torsional vibration of the rotating shaft is shown in Figure 3.

4.3. Case 3: Initial torsional vibration θ_0 = 0.03(rad)and initial angular velocity of $\dot{\theta}_0 = 1.0$ (rad/s)

From the parameters in Tables 1,2 and case 3.The author uses Maple software to simulate the torsional vibration of the rotating shaft is shown in Figure 4.

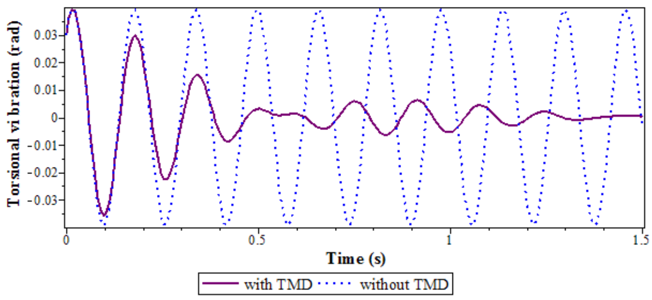

Figure 4. The vibration of the shart in the case with initial torsional vibration θ_0= 0.03(rad)and initial angular velocity of $\dot{\theta}_0 = 0,03$(rad)

From Figures 2, 3 and4. We find that the optimal parameters of the TMD are defined in this paper has a good effect for reducing torsional vibration of the rotating shaft.

5. CONCLUSION AND DISCUSSION

The main objective of this paper is to find the optimal parameters of the tuned mass damper to reduce torsional vibration for the shaft in the case of the rotating shaft with variable angular velocity. The optimal parameters of the tuned mass damper are determined by the maximization of equivalent viscous resistance method are expressed according to equations (21, 22). To evaluate the effect of reducing vibration, the author uses Maple software to simulate the torsional vibration of the whole system. Through vibration simulation, we find that the vibration amplitude of the rotating shaft is suppressed when the tuned mass damper is installed. This confirms that the optimal parameters of the tuned mass damper are found in this paper are reliable. Helping scientists easily find the optimal parameters when applying to eliminate torsional vibration of the rotating shafts with variable angular velocity.

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APPENDIX

Notation

m	Concentrated mass at the top of the TMD
k_m	Torsional stiffness of spring of the TMD
c	Damping coefficient of damper
L	Length of a pendulum of the TMD

k_t	Torsion spring coefficient of shaft
m_t	Mass of pendulum rod
ρ	Radius of gyration of primary system
M	Mass of of primary system
φ	Angular displacement of the shaft
φ_1	Angular displacement of rotor
φ_2	Relative torsional angle between TMD and rotor
θ	Torsional vibration of the primary system
θ_0	Initial condition of the torsional vibration angle
μ	Ratio between mass of the TMD and mass of primary system
α	Tuning ratio of the TMD
ξ	Damping ratio of the TMD
ω_D	Natural frequency of vibration of primary system
ω_d	Natural frequency of vibration of the TMD
ω	Frequency of angular acceleration of the rotating shaft
γ	Ratio between length of pendulum and radius of gyration of rotor

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THÔNG TIN TÁC GIẢ

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